Life of Fred

Trigonometry

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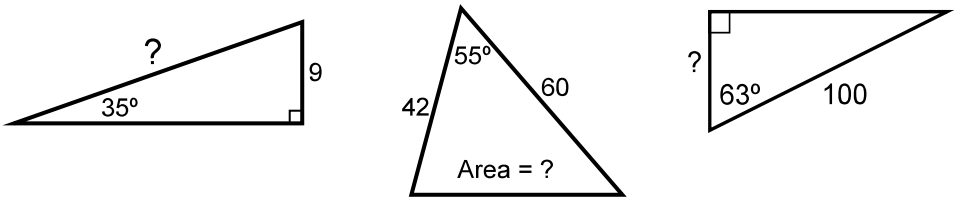
What's in Trig?

Trigonometry plays with triangles. Mostly right triangles. *Trigon* means triangle and *metry* means measuring (in Greek). Someone probably stuck the "o" in *trigon*-o-*metry* to make it easier to pronounce.

By the end of the first chapter of this book you'll be able to find the quantities indicated by a question mark:



and you'll know the first of the three major trig functions (the sine function). That's the first nine pages of the book.

The rest of the book pretty much flows naturally from those first nine pages. If you looked at the definition of the sine function in chapter one for several minutes, you could predict how the cosine and tangent functions would be defined in chapter two. The only thing you wouldn't know is their names.

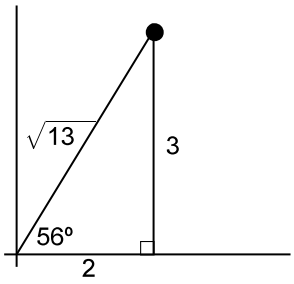
In chapters three through nine, we take the concepts of sine, cosine, and tangent and stretch them like taffy. In chapter one we were taking the sine of the acute angles in a right triangle. In chapter three we wonder what the sine of 110º would equal. In chapter four we find the basic algebraic facts about sine, cosine, and tangent, such as $(\text{sine of A})^2 + (\text{cosine of A})^2 = 1$ for every angle A. These basic facts will be used later in the book. In chapter five we invent a new way to measure angles. Instead of talking about 30º, we have $\pi/6$ radians.

Chapter six: We put sines, cosines, and tangents in algebra equations and solve them.

Chapter seven: The trig functions are used in triangles that are not right triangles.

Chapter eight: We turn the sine, cosine, and tangent functions inside out by finding their inverses. Back in algebra we knew that if h(Meddie) = apple pie, then the inverse function, h^{-1}, would give us h^{-1}(apple pie) = Meddie.

Chapter nine: We locate the point (2, 3) on a graph using angles and lengths instead of just lengths. Instead of saying that the point is two units to the right and three units upward, we'll say that it is roughly 3.6 units from the origin at an angle of approximately 56º.



The real surprise comes in the last chapter. All the chapters from two through nine you might have been able to predict, but not chapter ten. In that last chapter we stir together parts of what we've learned in trigonometry so far and come up with the answer to $\sqrt[5]{1}$. Not just the answer ($\sqrt[5]{1} = 1$) that you know from algebra. We arrive at five *different* answers. By the end of that final chapter you will be able to name five different numbers each of which, when raised to the fifth power, will equal one. Probably less than 2% of all college graduates can name those numbers.

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Chapter One

Sine

A strong wind blew from the south. That seemed like a good sign to Fred since he was heading north on Highway 135 in Kansas. It wouldn't be long before he'd be back at the university where he lived. *Home*—that had such a sweet sound to it.

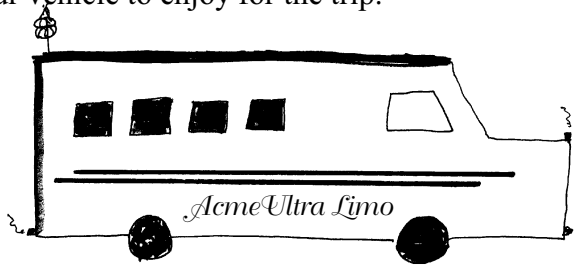
He thought back over the last several days. On Friday, his sixth birthday, he had been seized by the induction evaders investigators and had endured 27 hours chained inside a military prisoner transport. He spent the weekend as a soldier down in a U.S. Army camp in Texas. By Monday he had an honorable discharge. The army chaplain paid his bus fare so that he could get back home. On the bus he had made friends with George and Cheryl Mittens and their three daughters and the girls' four friends. The bus was hijacked and driven to Cuba, Kansas. And finally on Tuesday after another bus ride to the South Kansas library, George, who had become a multi-billionaire on this bus trip, ordered a limo for Fred's trip back to the northern part of Kansas.✶

The back of the limo was almost as big as Fred's office at KITTENS University (Kansas Institute for Teaching Technology, Engineering, and Natural Sciences). The interior was all leather and gold with a telephone, a television, and a wet bar.

The windows were tinted a dark blue-gray, making it difficult for Fred to see the evening sky.

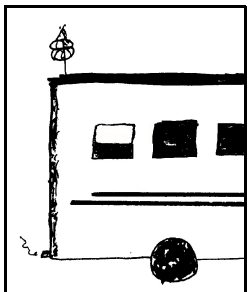
"Is it okay if I roll down the window to look outside?" Fred asked the driver.

"Of course, Sir," the driver responded. "You may do as you wish. This is your vehicle to enjoy for the trip."



✶ The adventures of Fred from Thursday afternoon (the day before Fred's sixth birthday) to Monday are told in *Life of Fred: Beginning Algebra.* Monday and Tuesday are chronicled in *Life of Fred: Advanced Algebra.*

Fred rolled down the window and was instantly sorry. His lap was filled with snow.



Classic mistake during snowy weather

The bus driver put the limo on AUTOMATIC PILOT and raced back to assist his young passenger. "Don't worry about it, Sir," the driver assured him. "This happens frequently." He vacuumed Fred's lap with a wet/dry vacuum. "Perhaps you would enjoy a bit of dinner before we arrive at KITTENS?"

When Fred looked down at his lap, the driver thought Fred was nodding "yes," and so he began dinner preparations in the limo's kitchen. Fred was hoping that dinner would be a small slice of pineapple pizza. That would hit the spot before he arrived back at his office and could visit the vending machines down the hall. He had recently made it his goal to weigh 40 pounds before he hit puberty. That would mean that he would have to increase his body weight by 10% in the next seven years. The driver/chef placed a large oak table in front of Fred and brought in what he had called "a bit of dinner":

Appetizer

Escargots in an Applewood-smoked Bacon Sauce

Soup

Spring Pea and Squash Blossom Soup with Duck Foie Gras

Salad

Montrachet Goat Cheese Melted over Young Field Greens

Pasta

Smoked Pheasant Ravioli with Fresh Tarragon

Sorbet✶

Apple-Lime

Entree

Saltimbocca of Salmon in a Bed of Northern Elk Medallions

Dessert

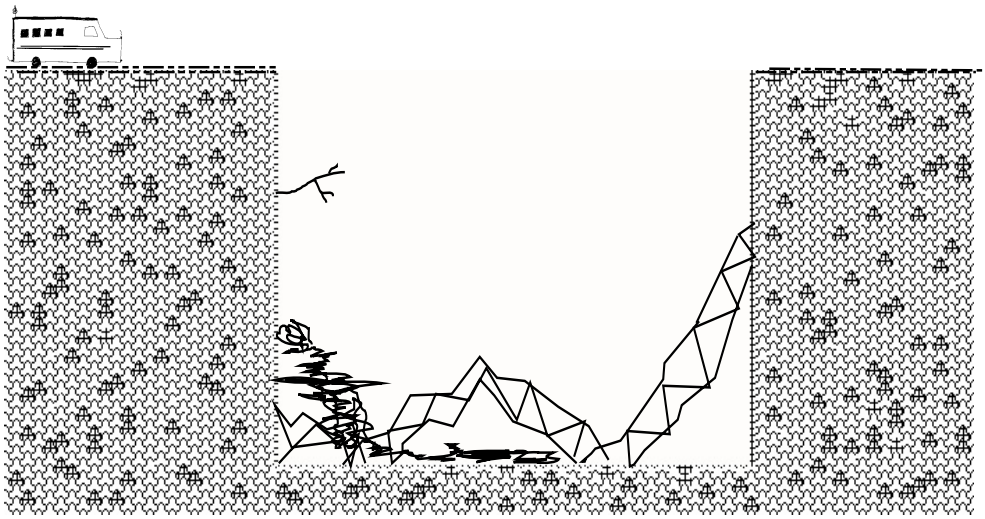
Jell-O®

✶ A sorbet is a fruit or vegetable ice which is served before the main course as a palate cleanser. *Webster's Ninth New Collegiate Dictionary* indicates that the correct pronunciation is SOAR-bet. This reflects the fact that sorbet is a word originally from Turkish. *Webster's Tenth* switches to soar-BAY which is the way most people seem to pronounce it today.

Fred sat there stunned. The vending machines down the hall never had anything like this. He carefully nibbled some of the young field greens (avoiding the melted goat cheese).

Suddenly the limo rolled to a stop. The driver looked at his watch and said, "This is too early for the limo to be coming to a stop. We've got another ten minutes before we get to KITTENS." He raced to the front to see why the AUTOMATIC PILOT had stopped the car.

The Troubled Waters Canyon Bridge had completely collapsed. In front of the car was a chasm about a hundred feet across.



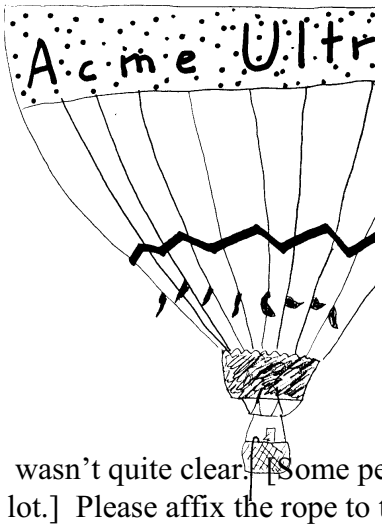
Fred and the chauffeur got out of the car and looked at the mess. They could see the lights of the university in the distance.

"Don't worry, Sir," the driver assured Fred. "Acme Ultra Limo Service guarantees that we'll get you to your destination. It's our Gold Service.* "

The driver headed to the trunk of the car and pulled out a large wooden box marked, "Canyon-Fording Emergency #351." Fred watched the driver unpack and inflate a large hot-air balloon.

"If you will just climb in, Sir," the driver said as he lifted his 37-lb. passenger into the balloon's basket.

✶ Acme Ultra Limo is often abbreviated as AU Limo. In chemistry, Au is the symbol for gold.



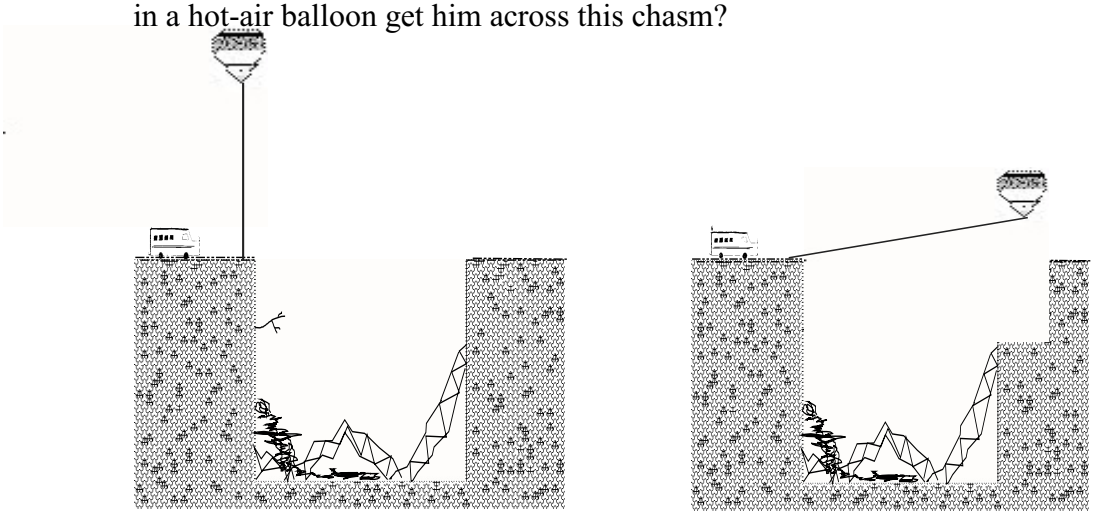
"But, but, but," Fred exclaimed. "I don't know how to fly one of these things!"

"That's quite all right, Sir," the driver answered. "If you would please toss one end of the rope out of the basket and secure the other end, then everything will be quite safe."

Fred did as he was asked. He opened the package marked "Canyon-Fording Emergency Balloon-Tether 120-foot Rope #351A", tossed one end out of the basket and tied the other end around his waist.

The driver looked at what Fred had done and gasped. "Oh no, Sir! I'm afraid I wasn't quite clear. [Some people who serve use the word *quite* quite a lot.] Please affix the rope to the basket. I shall attach the other end to this stake in the ground. Then as the balloon ascends, you shall never be more than 120 feet from me."

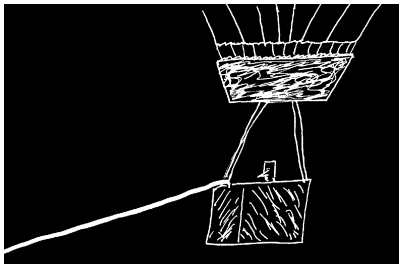
Fred couldn't figure out what was going on. How could going up in a hot-air balloon get him across this chasm?



What Fred thought was going to happen

What really happened

(The driver had read the first seven words of this chapter.) He called out to Fred, "You may jump out of the basket now, Sir."



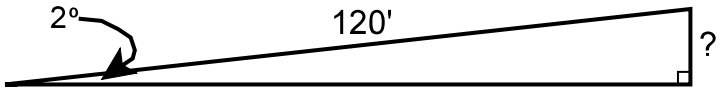
Jump? thought Fred. Where's my parachute? I really can't see how far it is to the ground. It's too dark.

"Driver," Fred called out in the darkness. "How far am I from the ground?"

"I can't tell, Sir," he responded. "It's too dark and you're too far away."

Fred said, "I know the rope is 120 feet long. Can you tell me what my **angle of elevation** is?"

"Very good, Sir." The driver headed to the storage unit in limo and grabbed a protractor,* put it on the ground and measured Fred's angle of elevation (which is how far above the horizontal he was). "Sir," he called to Fred, "Your angle of elevation is two degrees."



Fred needed to know how far the balloon was off the ground (marked by a "?" in the diagram). We have now arrived at the heart of trigonometry. Trig deals with the angles and sides of right triangles.

(Your reading speed should be adjusted appropriately since you're in a more mathematical section. For example, let's read the three sentences of the previous paragraph and see what we find.

When you read that Fred needed to find the value of the "?," the thought may have come to you as you looked at the diagram that nothing in algebra or in geometry ever showed you how to find the length of the side that is opposite the 2° angle. The sentence, "We have now arrived at the heart of trigonometry," is really a most amazing statement. Five pages into the first chapter and we have a practical example of the use of the first trig function (the sine function) and by page six we will have defined it. No other trig textbook that I know of gets to this point this quickly. One trig book takes 165 pages to get to its first application of the sine function. The third sentence, "Trig deals with the angles and sides of right triangles," is one that you may have used your highlighter on—assuming this is your book and not someone else's.)

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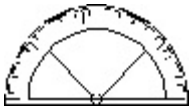
* Protractors are angle-measuring devices. They're usually plastic. You won't need one for trig, but you should have one if you're running a limousine company that offers Gold Service.

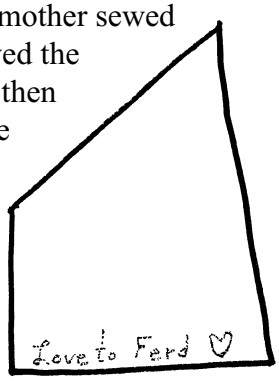


Then Dr. Speck asked the wrong question: "Tell me your thoughts boy. Tell me what you obsess about."

Maybe this doctor isn't so bad after all, Fred thought. I really don't think about my stomachache that much. What I have really been turning over in my mind—should I tell him? "You really want to know? You really care?" Fred asked. "It's been on my mind day and night recently."

Clearly fixated went into the patient's medical file.

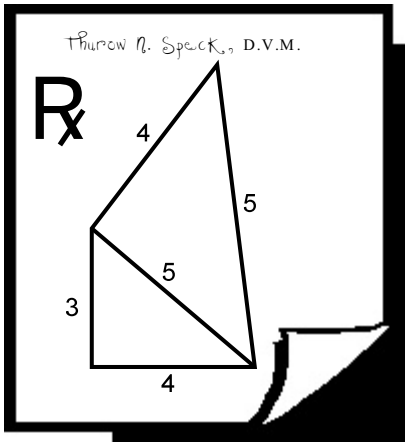
"It all started with this handkerchief that my mother sewed for me." Fred withdrew it from his pocket and showed the doctor. "She spent a week sewing this for me. And then on the bottom she embroidered this message. It's the only thing I have from my mother."



Speck wrote, oedipal...exhibits rare handkerchief fetish. Also mother can't sew straight and is dyslexic.

"As I looked at this gift, I realized it might be one of the most perfect law-of-cosines problems that has ever been invented. I've never seen a trig textbook with such a knotty nut to crack. Here are the measurements."

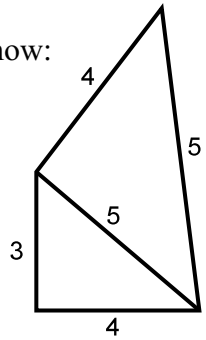
Fred took Speck's prescription pad and wrote:

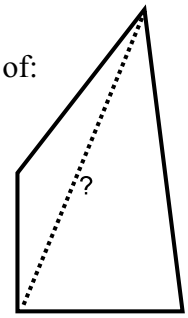


"These are the measurements of edges of the handkerchief and the length of one of the diagonals," Fred said. He was drifting into full lecture mode. "The problem is to find the length of the other diagonal."

"Well, son, that shouldn't be too hard. I got a ruler right here in my desk." He opened the drawer and removed some old racing forms, an unpaid parking ticket, a pair of gold-tooth cuff links, and a used hypodermic needle. "I don't seem to have a ruler here. I'll be back in six minutes." He left the room and headed down the hall to the break room.

While Dr. Speck is taking another smoke break, it might be fun to try and solve the handkerchief problem.

You know: 

and you want to find the length of: 

When Fred calls a problem "knotty," you know that it is one that might stump some trigonometry teachers. But this problem is also a *fair* problem. It can be solved using only the material we've covered so far in this book. (". . . in this *book*," not ". . . in this chapter." You may be using material from previous chapters.)

If you really, really want to learn trigonometry and you enjoy a challenge, this problem is for you. Its solution involves several steps. About one student in a hundred in Fred's trig classes can find the length of the other diagonal, which is (roughly) 6.069. Regardless of whether you succeed, it is worth the effort to try. The Handkerchief Problem is good exercise for the brain and will make it stronger.

> Far better it is to dare mighty things, to win glorious triumphs, even though checkered by failure, than to take rank with those poor spirits who neither enjoy much nor suffer much, because they live in the gray twilight that knows not victory nor defeat. —Theodore Roosevelt

Teddy was obviously talking about the Handkerchief Problem.

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