Zillions of Practice Problems

Advanced Algebra

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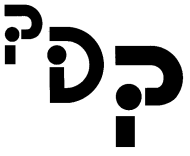

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What This Book Is All About

The first practice book, *Zillions of Practice Problems for Beginning Algebra*, was well received. It has served a definite need for many students of beginning algebra.

With that encouragement, I have spent months creating this practice book for advanced algebra.

In *Life of Fred: Advanced Algebra*, there are *Your Turn to Play* sections after each topic. And each *Your Turn to Play* offers completely worked out solutions to each question.

At the end of each of the ten advanced algebra chapters are six problem sets.* In addition, there are five "Looking Back" chapters that review beginning algebra. The answers to all these questions are either given in the book or in the study guide, *Fred's Home Companion: Advanced Algebra.*

In addition, *Fred's Home Companion: Advanced Algebra* offers additional problems together with their answers.

The book you are now holding, *Zillions of Practice Problems Advanced Algebra*, contains a massive number of new exercises—all keyed to *Life of Fred: Advanced Algebra.*

As you work through each chapter in *Life of Fred: Advanced Algebra*, you may do as many of the problems as you like in the corresponding chapter in this book.

Each of the problems is fully worked out. Some of the discussions are more than a page long.

There are nine examples of radical equations (like $\sqrt{x^2 - x + 16} = 6$). More than enough to learn how to solve them.

There are eleven completely worked out examples of linear programming (like this word problem: There were a lot of rocks in my backyard that I needed to remove. I had two options. I could either use a hammer and wheelbarrow or use dynamite and a truck.

With the hammer and wheelbarrow, my cost would be $1/day and I would get injured about 0.2 times each day. (On the average, I would get injured once every 5 days.)

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* Quick math. Ten chapters × 6 problem sets/chapter = 60 problem sets

With the dynamite and a truck, my cost would be $50/day and I would get injured about 0.1 times each day.

I had $817 to spend on the project and I could experience at most 5 injuries. (After five injuries I would quit and do something a lot less dangerous.)

I can remove 4 tons of rocks each day using hammer and wheelbarrow and 20 tons using dynamite and a truck. What is my best course of action in order to remove as much rock as possible?) Linear programming word problems look hard to many students. After you have done four or five of them, they start to look easy. After you have done eleven of them (in this book) and all the ones in *Life of Fred: Advanced Algebra* and in *Fred's Home Companion: Advanced Algebra*, they become boringly easy. (That's our goal!)

Many other second-year algebra books (called Advanced Algebra or Algebra 2) contain *zero* linear programming problems.

Some also skip:

- permutations
- matrices
- sigma notation
- math induction proofs
- partial fractions, and
- the change-of-base rule for logs

Both this book and *Life of Fred: Advanced Algebra* cover all of these topics. They belong in any decent second-year algebra course.

Many other math publishers leave out these topics. The results are:

1) The students don't realize that the topics have been omitted.

2) The students finish the book more easily.

3) Everything is fine until they hit their SAT exams or later math courses. Then they find out the truth.

CHAPTERS IN THIS BOOK

Each advanced algebra chapter in this book is divided into two parts.

✯ The first part takes each topic and offers a zillion problems.

✯ The second part is called the Mixed Bag. It consists of a variety of problems from the chapter and review problems from the beginning of the book up to that point.

ELIMINATING TEMPTATION

The solutions and answers are all given in the back half of the book. The first question in this book is numbered "35." The second one is "101." In most ordinary books, they are numbered, "1, 2, 3 . . ." which is really silly when you think about it. In those books, when you look up the answer to "1" you might accidentally see the answer to "2" and that would spoil all the fun.

It usually takes three or four seconds (I timed it) to locate a solution in the back of the book.

YOUR FUTURE

This is the last bit of high school algebra that you will need. *Life of Fred: Advanced Algeba*, together with its study guide, *Fred's Home Companion: Advanced Algebra*, cover the material in 101 daily lessons. If you tuck in a lot of practice with this book, you might fluff things up to 140 or 160 days. Or, horrors, 180 days! But that is still less than half a year.

After advanced algebra, high school geometry, and a course in trigonometry, you have all the high school math required by virtually every major university.

With my best wishes for your future,

Stan Schmitt

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301.

My dog Wufwuf has 4 fleas on the day I have given her a bath. We'll call that day #1. On the next day (day #2) she has 28 fleas and the day after that (day #3), 196 fleas. The sequence is 4, $4 \cdot 7$, $4 \cdot 7^2$, $4 \cdot 7^3$, ...
(4·7 means 4 times 7)

How many fleas does Wufwuf have on day #22?

330. Most dogs like to fetch balls. Wufwuf is different. She likes to fetch pumpkins. One Halloween she ran around the neighborhood and stole 3 pumpkins and brought them home. She didn't tell me about her theft but hid the pumpkins behind the backyard wall.

On the second Halloween, she stole 12 pumpkins and added them to her stash behind the backyard wall.

On the third Halloween, she stole 48 pumpkins. Each year she got better at finding pumpkins and taking them. Each year she took 4 times as many as the previous year.

How many pumpkins were hidden behind the backyard wall after 8 Halloweens? (In case you are wondering, Wufwuf only stole plastic pumpkins. They didn't rot.)

Sigma Notation Σ

$$\sum_{i=1}^{5} 8ix = 8x + 8(2)x + 8(3)x + 8(4)x + 8(5)x$$

$$\sum_{i=1}^{3} 7x^i = 7x + 7x^2 + 7x^3$$

77. Write out what each of these means:

$$\sum_{i=1}^{8} 5i$$

$$\sum_{i=1}^{4} \log(x + i)$$

$$\sum_{i=1}^{3} 8x^i y^{i+3}$$

329. Prove $1 + 3 + 5 + 7 + \ldots + 2n - 1 = n^2$ for every natural number n.

The first step is to prove the n = 1 statement is true. This part of the math induction proof is usually super obvious.

$n = 1 \Rightarrow 1 \stackrel{?}{=} 1^2$ I told you it was obvious.

The second step is to assume the n = k statement is true.
Namely, we assume $n = k \Rightarrow 1 + 3 + 5 + \ldots + 2k - 1 = k^2$.
We are allowed to use this fact in trying to prove the n = k+1 statement.

To prove: $n = k + 1 \Rightarrow 1 + 3 + 5 + \ldots + 2(k + 1) - 1 \stackrel{?}{=} (k + 1)^2$

Let's start with what we are assuming to be true:

$$1 + 3 + 5 + \ldots + 2k - 1 = k^2$$

What's the next number after 2k – 1 in the series? These are odd numbers that are separated from each other by 2. If I add 2 to any number in the series, I get the next number.

The next number after 2k – 1 is 2k – 1 + 2.

2k – 1 + 2 simplifies to 2k + 1.

If I add 2k + 1 to both sides of the series I'm assuming to be true, I get:

$$1 + 3 + 5 + \ldots + 2k - 1 + \mathbf{2k + 1} = k^2 + \mathbf{2k + 1}$$

But this is exactly what I'm trying to show is true.

The details: 2k + 1 is the same as 2(k + 1) – 1

$k^2 + 2k + 1$ factors into $(k + 1)^2$

Done!

330. Most dogs like to fetch balls. Wufwuf is different. She likes to fetch pumpkins. One Halloween she ran around the neighborhood and stole 3 pumpkins and brought them home. She didn't tell me about her theft but hid the pumpkins behind the backyard wall.

On the second Halloween, she stole 12 pumpkins and added them to her stash behind the backyard wall.

On the third Halloween, she stole 48 pumpkins. Each year she got better at finding pumpkins and taking them. Each year she took 4 times as many as the previous year.

How many pumpkins were hidden behind the backyard wall after 8 Halloweens?

a = 3, r = 4, n = 8

$$s = \frac{a(1 - r^n)}{1 - r} = \frac{3(1 - 4^8)}{1 - 4} = \frac{3(1 - 65536)}{-3} = 65{,}535 \text{ pumpkins.}$$

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