

*Fred's Home Companion*  
*Beginning Algebra*

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Polka Dot Publishing

## *What is Fred's Home Companion?*

It is lots of things. Since *Life of Fred: Beginning Algebra* was first published, there have been requests from home schoolers, teachers, and adults who are learning beginning algebra. This book is a response to those needs.

Need #1: I'm a home schooler and I would like my algebra chopped up into daily bite-sized pieces.

Done! *Fred's Home Companion: Beginning Algebra* offers you 108 daily readings. In the space of one summer, for example, you can finish all of beginning algebra and still have plenty of time to do other things.

Need #2: I'm a classroom teacher and I want someone (like you!) to write out all my lesson plans and lecture notes.

Done! Here are 108 lesson plans. Each one tells you what part of the book you'll be covering. Each one gives you your lecture notes—problems to present at the board (along with their solutions).

Need #3: I'm an adult working my way through your LOF:BA book and I'd like the answer key for the end-of-the-chapter problems. In the book you give the answers to half the problems.

Done! Here is the answer key.

Need #4: I'm in need of a lot of practice in algebra. Although you have a lot of problems for me to work on in LOF:BA, I want a bunch more. I really want to pound it into my head.

Done! In this book we supply a ton of additional beginning algebra problems. Finish all of these problems in addition to the ones in *LOF:BA*, and you should be able to join Fred as a professor of mathematics at KITTENS University.

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# Lesson One

## Finite/Infinite, Exponents, and Counting

*Life of Fred:*  
*Beginning Algebra*  
pp. 15–16

1. Are there a finite or infinite number of grains of sand on all the beaches in the world?
2.  $10^{79}$  means 10 times 10 times 10 . . . seventy-nine times. What does  $3^4$  equal?
3. Which is larger:  $2^5$  or  $5^2$ ?
4. In Fred's dream the set (collection) of roses was infinite. The set of even natural numbers,  $\{2, 4, 6, 8, 10, 12, 14, \dots\}$ , is infinite. The set of all possible melodies is infinite.

You don't find infinite sets at the grocery store.

You don't find infinite sets in your laundry basket.

Where *is* a good place to find infinite sets?

5. What does  $1^{8369}$  equal?
6. What would you multiply  $10^2$  by in order to get  $10^5$ ?

### Intermission

Some people like to argue that infinite sets don't really exist. After all, they say, they're just a figment of your imagination. It's all in your head."

By that same argument I could prove that pain doesn't exist. When you cut your finger, it's in your brain that the pain is experienced.

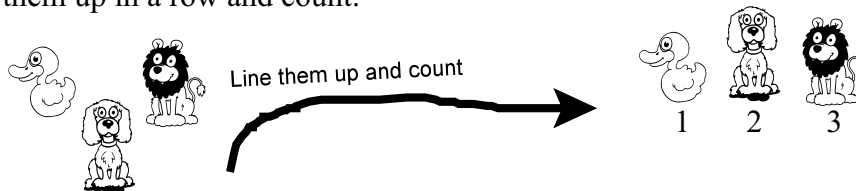
And the pleasure of a bite of warm pizza doesn't exist.

And the number three doesn't exist.

And truth doesn't exist.

Just because it is happening inside your skull doesn't mean that it doesn't exist.

7. When you want to count something, one of the easiest ways is to line them up in a row and count.



A hard question: Why doesn't it make a difference which order you line them up? Why do you always get the same answer?

## ANSWERS

1. Nothing in the physical universe is infinite. There are a finite number of grains of sand.
2.  $3 \times 3 \times 3 \times 3$  which is 81.
3.  $2^5$  is  $2 \times 2 \times 2 \times 2 \times 2$  which is 32.  $5^2$  is  $5 \times 5$  which is 25. So  $2^5$  is larger.
4. The set of roses in a dream, the set of even natural numbers, and the set of all possible melodies are all things that we can conceive. They are not things we can touch. To find infinite sets, one of the best places to look is your mind.
5. If you keep multiplying 1 times itself, you will always get an answer equal to 1.
6.  $10^2 \times ? = 10^5$  is a restatement of the question.  
 $100 \times ? = 100,000$   
 $100 \times 1000 = 100,000$   
 $10^2 \times 10^3 = 10^5$
7. Wow. That is something that most people never think about. They would say that it's *obvious* that the way you line up the items won't affect how many there are.

Could it be that it's obvious to them because that's what they've always experienced? But suppose the world were created a little differently. Suppose that the order in which you lined up the objects affected how many there were? Then everyone would go around saying that it's obvious that the way you line up objects affects how many there are. One of the enduring mysteries of mathematics is how well the stuff that goes on in our heads reflects what goes on out there in the "real world." *That didn't have to happen.*

One of the fun things I sometimes do in a calculus class (when we're studying infinite series) is to write on the board:

$$1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + \dots$$

and ask the students what the sum is.

If I add together the pairs I get  $(1 - 1) + (1 - 1) + (1 - 1) + \dots$  which is  $0 + 0 + 0 + \dots$  which equals zero.

If I combine the second and third numbers together, the fourth and fifth numbers together, etc., I get  $1 - \underline{1 + 1} - \underline{1 + 1} - \dots = 1 + 0 + 0 + \dots$  which equals one.

## Lesson Two

### Natural Numbers, Whole Numbers, Parentheses, Braces, and Brackets

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1. Match each item in the left column with one item in the right column.

[ ]	parentheses
( )	braces
{ }	brackets

*Life of Fred:  
Beginning Algebra  
p. 17*

2. When you list the elements of a set, you enclose them in braces. Write the set that contains your three favorite numbers.
3. Write the set that contains the four smallest natural numbers.
4. Is it possible to name a natural number that is not a whole number?
5. This set has three elements in it:  $\{7, 4, 0\}$ .

This set has one element in it:  $\{5\}$ .

This set has one element in it:  $\{0\}$ . Zero is *something*. It's not nothing. Zero is a number. On the other hand, if you have a paper bag with zero things in it, there is *nothing* in the bag.

Write the set that has zero elements in it.

6. In algebra, when you put parentheses (or brackets) around some expression, you are saying, "Do me first."

For example,  $(3 + 4) + 9$  means that you would add together the 3 and the 4 first.....  $7 + 9$   
and then..... 16.

$(3 + 4) + 9$  gives an answer of 16. Evaluate  $3 + (4 + 9)$ .

7. One of the fanciest laws of algebra is the **distributive law**. It states that for any three numbers,  $a$ ,  $b$  and  $c$ , it is always true that

$$a(b + c) = ab + ac$$

Evaluate  $7(9 + 12)$  and evaluate  $7 \times 9 + 7 \times 12$  and see if they come out to the same answer.

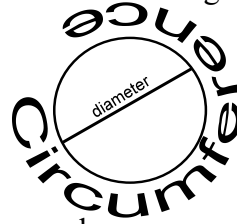
8. What does  $6^2$  equal?
9. Harder question: Write the set whose only element is the null set.

## ANSWERS

1.                      [ ] ..... ➤ parentheses  
                           ( ) ..... ➤ braces  
                           { } ..... ➤ brackets

2. Your answer will probably be different than mine. The set containing my three favorite numbers is  $\{21, 0, \pi\}$ .

Note:  $\pi$  is a number. It is the number you get when you divide the circumference of a circle by its diameter. It is approximately equal to 3.14159.



3.  $\{1, 2, 3, 4\}$   
 4. It isn't possible since every natural number is a whole number.  
 5.  $\{\}$  This is sometimes called the **null set** or the **empty set**.  
 6.  $3 + (4 + 9)$   
 $= 3 + 13$   
 $= 16$

So  $(3 + 4) + 9$  equals  $3 + (4 + 9)$ .

It turns out that this is true for any three numbers. If  $a$ ,  $b$  and  $c$  are any three numbers, then it is always true that  $(a + b) + c = a + (b + c)$ . This is called the **associative law of addition**.

- |                |                            |
|----------------|----------------------------|
| 7. $7(9 + 12)$ | $7 \times 9 + 7 \times 12$ |
| $7(21)$        | $63 + 84$                  |
| $147$          | $147$                      |

8.  $6^2 = 6 \times 6 = 36$  [This is a problem from the previous lesson.]  
 9.  $\{\{\}\}$  The null set is  $\{\}$ . I have placed that set within braces to create a set whose only element is the null set.

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